A STUDY OF A MODIFIED BINARY SEARCH FOR USE IN SENSITIVITY TESTING

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THESIS

A STUDY OF A MODIFIED BINARY
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by

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Search for Use in Sensitivity Testing

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ABSTRACT

A modified binary search routine was developed by Tysver to provide inputs (estimates of parameters of threshold stimulus distributions) to the Probit and Staircase techniques for sensitivity testing. A high speed digital computer was used to provide simulated sensitivity data in order to test the modified search (more specifically, study the parameter estimates generated by the search).

The results show that the modified search does not give practical input information to the techniques mentioned above. However, further refinements of extensions to the basic search patterns should yield better estimates of the distribution parameters.

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I. INTRODUCTION

The estimations of parameters describing distributions of outcomes in various physical situations are usually easy to compute and have known accuracy. The best estimate of the mean time to failure of a light bulb, for example, is simply the average failure time of a sample of light bulbs. The best estimates of the mean and variance of a normally distributed outcome are the sample average and sample variance using well known formulas. The samples in these cases are made up of a collection of point estimates of the population mean.

However, there exists a class of physical situations in nature for which the parameters of the distributions cannot be estimated by the usual point estimate techniques. Examples are the threshold stimulus to detonate an explosive charge, the amount of insecticide necessary to kill a pesky mosquito, and the intensity of a light source necessary for visual perception at a given range. These examples share a common characteristic: they all deal with an application of some form of impulse stimulus. The study of this class of physical situations has been labeled "sensitivity analysis."

When applying a stimulus to a certain sense, one of two responses can occur. The subject has a positive response (e.g., detonates, dies, sees the light source) or a

negative response (e.g., fails to detonate, lives, fails to see the light source). The response is simply a Bernoulli random variable and the probability of a response varies with the stimulus level applied.

Major efforts in the past have been devoted to statistical techniques centered on the density function and point estimates. However, estimates of parameters of sensitivity distributions cannot be established by usual point estimation methods since data realized in sensitivity testing is obtained from distribution functions, not density functions.

A few techniques have been developed over the years to accomplish the task of providing parameter estimates from sensitivity data. These include the Probit technique, as described by Finney [2], the Staircase or Bruceton method reviewed by Dixon and Mood [1], the Countback method of Lewis [4], and others. Many methods are devoted to finding the 50% level (the level at which one would expect 50% positive responses to the stimulus [4]). Others claim to estimate parameters of sensitivity distributions accurately but require knowledge or assumptions about the parameters prior to testing.

The Modified Binary Search (MBS) is a method designed by Tysver [5] to examine physical situations for which sensitivity testing is applicable. Various levels of stimuli are used as a basis for computing estimates of parameters of the appropriate threshold stimulus distribution.

Tysver proposed that the estimates from MBS be used as inputs to previously mentioned tests (e.g. Probit, Staircase) and also in predictions for safety and reliability.

The results of [5] were examined and tested through the use of simulation conducted on a high-speed electronic computer. Standard Monte Carlo techniques were used to generate the required sensitivity data.

II. THE MODIFIED BINARY SEARCH

A. UNDERLYING SENSITIVITY MODEL

A brief description of the model used in the Modified Binary Search (MBS) is as follows. Let x be an applied stimulus level ($x \varepsilon (0,\infty)$). Define $Y = f(x)$ to be a Bernoulli random variable with realizations $y = 1$ for a positive response and $y = 0$ for no response. Next, define a response function $p(x)$ where

$$p(x) = \mathrm{Prob}(Y = 1|x),$$

and assume that $p(0) = 0$ and $p(\infty) = 1$. An investigator, however, should be able to determine a shorter interval (a,b) with $0 \leq a \leq x \leq b < \infty$ where $p(a) \approx 0$ and $p(b) \approx 1$ with a high degree of confidence. The response function is graphed in Figure 1.

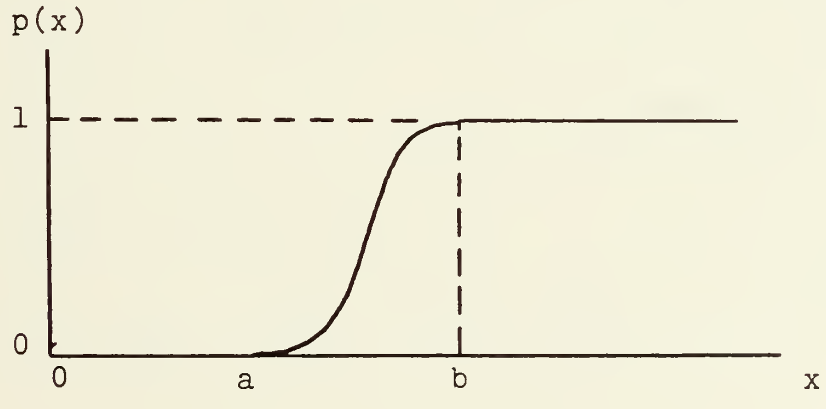


Figure 1. The Response Function

In other words, $p(x)$ is the cumulative distribution function for a random variable X (the threshold stimulus), where

$$p(x) = \text{Prob}(X \leq x).$$

If the applied stimulus level is greater than the threshold stimulus, X, then the response will be $y = 1$. If $x < X$, then the response will be $y = 0$. Note that

$$\text{Prob}(Y = 1|x) = \text{Prob}(X \leq x) = p(x)$$

and

$$\text{Prob}(Y = 0|x) = \text{Prob}(X > x) = 1 - p(x).$$

The threshold stimulus, X, or an appropriate transformation of the same, is assumed to be normally distributed. The validity of this assumption does not appear to be critical in the vicinity of the 50% response level, especially when the investigation is limited to small samples [6].

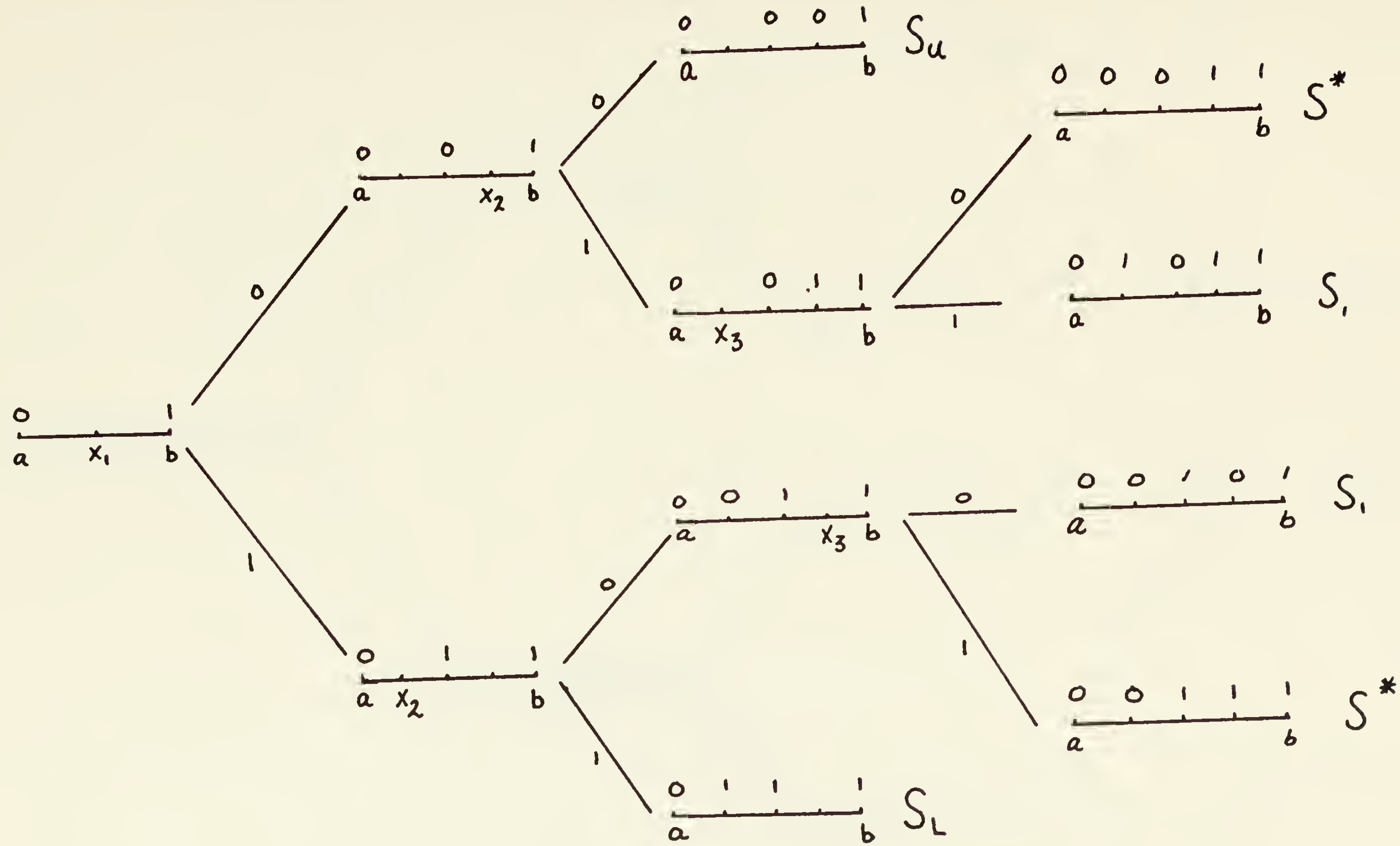
With the sensitivity model defined, the investigator is now faced with the task of preselecting the applied stimulus levels $(x_1, x_2, \ldots, x_n)$ or determining a precedure to choose each stimulus level x_{i+1} based upon previously tested stimulus levels $(x_1, x_2, \ldots, x_i)$ and the observed results $(y_1, y_2, \ldots, y_i)$. Once a sampling procedure has been selected, he conducts the investigation. The sample data $(y_1, y_2, \ldots, y_n)$ he obtains contain information on the distribution function of the threshold stimulus. Each response (y_i) tells the investigator whether or not the random threshold stimulus (X_i) was greater than the

applied stimulus level (x_i). The investigator is thus faced with estimating parameters of the threshold stimulus' density function from data describing the distribution function.

B. DESCRIPTION OF MBS

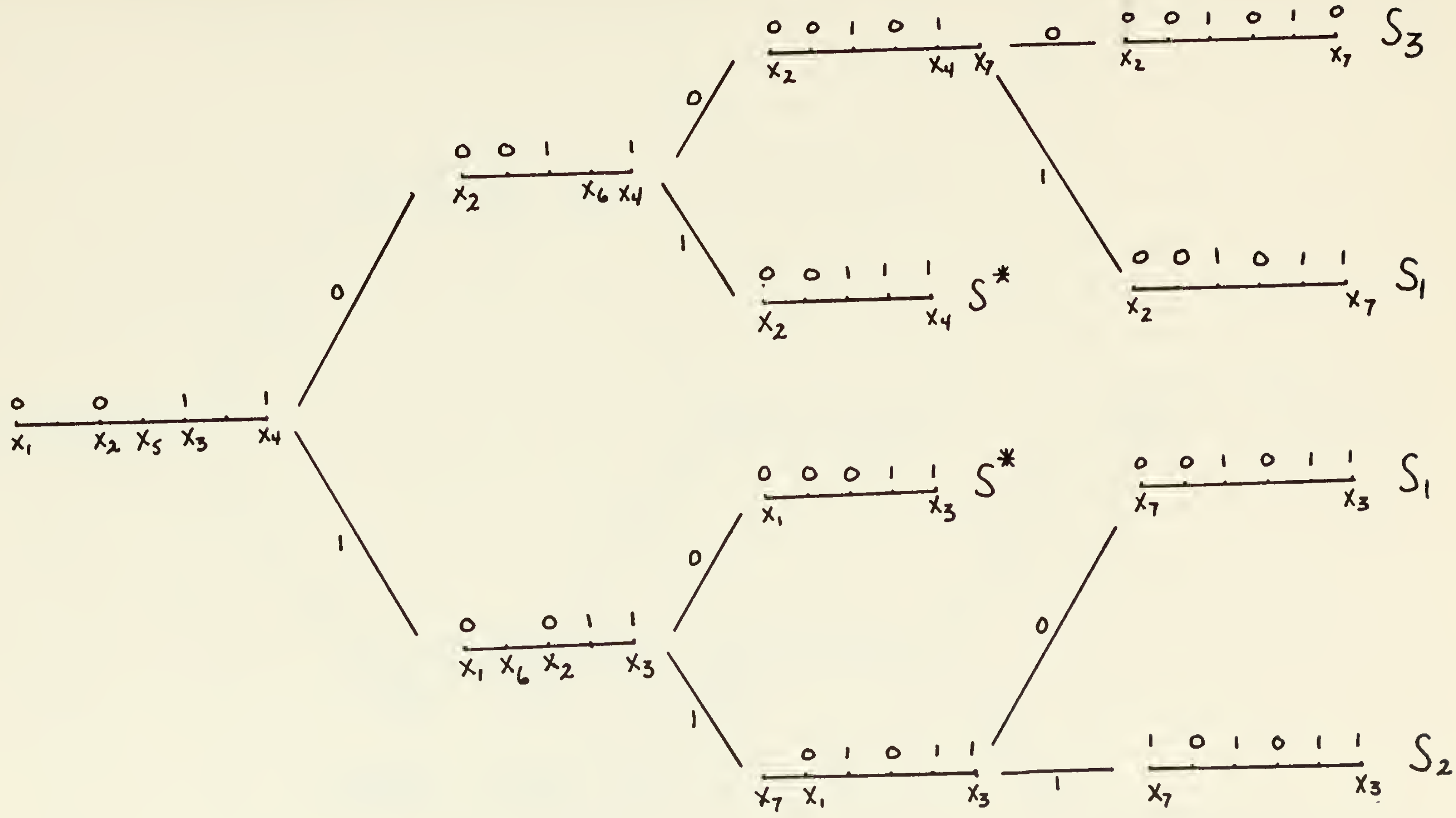
The investigator's first task is to choose reasonable endpoints a, and b over the range of stimulus levels where $p(a) \approx 0$ and $p(b) \approx 1$. Sampling outside the interval (a, b) will be unnecessary if the endpoints are chosen properly. For his first trial he selects $x_1 = (a + b)/2$. If $y = 1$, then he selects $x_2 = (a + b)/4$ for his next trial. If $y = 0$, however, he selects $x_2 = 3(a + b)/4$. The complete search procedure is diagramed in Figures 2 through 5. If it appears that the search is converging to one of the endpoints, then the investigator should question his initial choice of that endpoint.

As in other sensitivity analysis techniques, MBS does not terminate until an "inversion" occurs ($y_i = 1$ and $y_j = 0$ for some i, j where $x_i < x_j$). Six equidistant levels are tested in the vicinity of the inversion. Let the lowest of the six levels be denoted by x_L and the highest by x_u. Define the interval (x_L, x_u) as the "primary region" and the distance separating any two adjacent levels in this primary region as "DELX." Any levels tested outside of the primary region are separated by at least DELX. The preliminary search ends with one of three possible sequential outcomes, S_1, S_2, or S_3. The number of



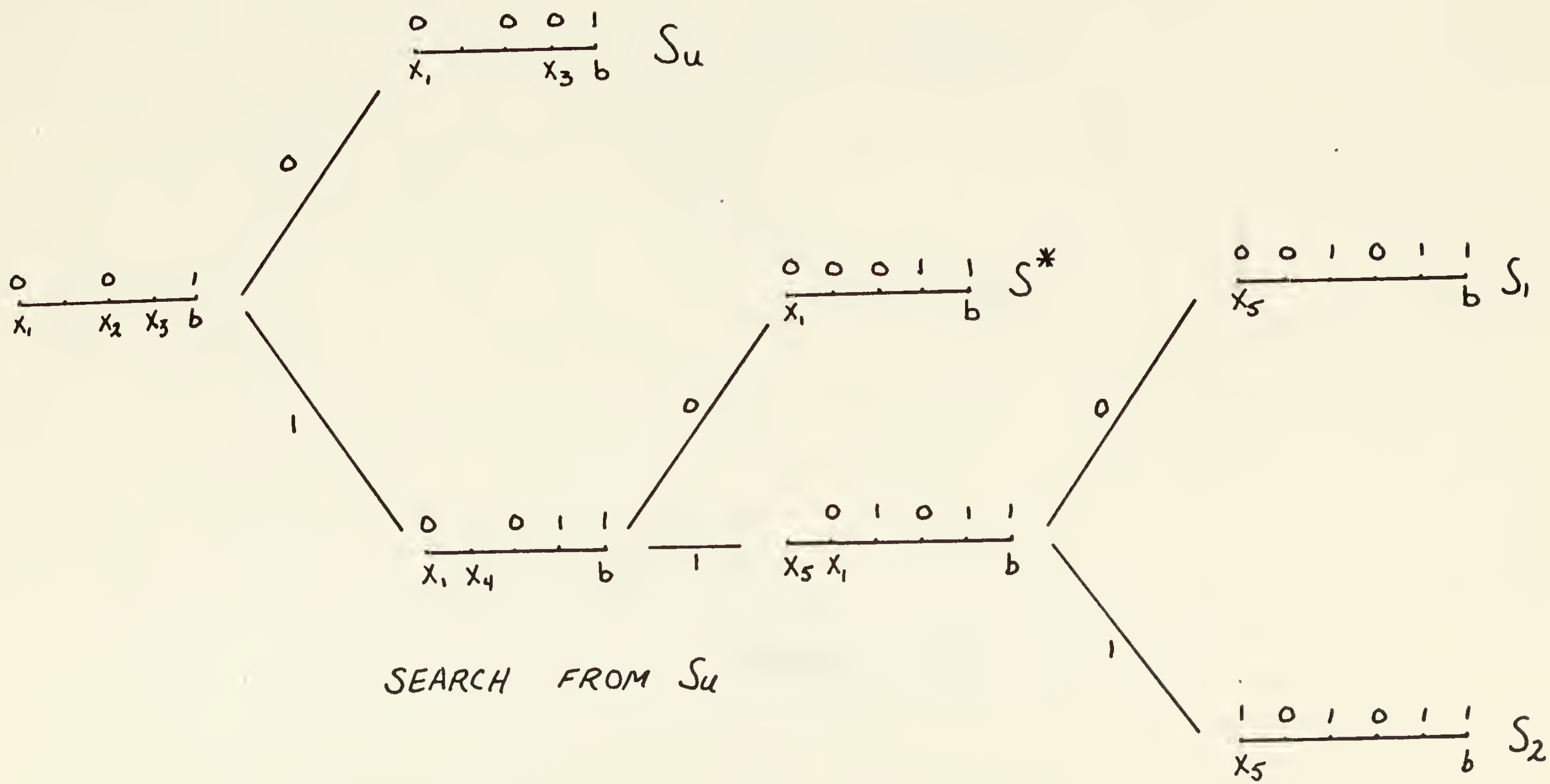
STARTING SEQUENCE

FIGURE 2



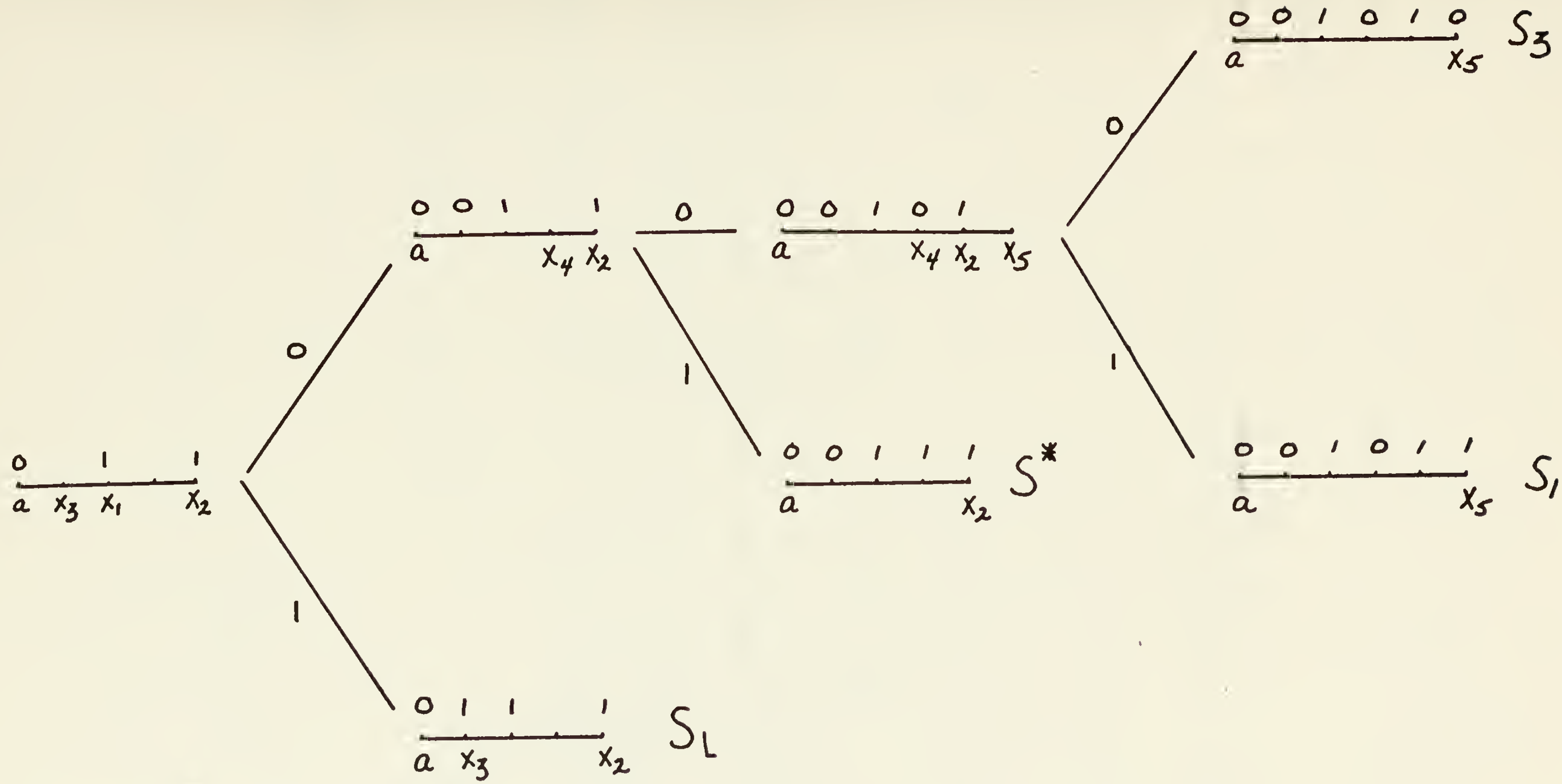
SEARCH FROM S*

FIGURE 3



SEARCH FROM Su

FIGURE 4



SEARCH FROM S_L

FIGURE 5

trials necessary to produce a sequence may vary with each experiment, but the primary region will consist of only three possible outcomes as depicted in Figure 6. All levels tested below the primary region yielded "0" responses while those tested above the region yielded positive responses.

$$S_1: \quad \frac{y = 0\ .\ .\ .\ .\ .\ 0\ 0\ 1\ 0\ 1\ 1\ .\ .\ .\ .\ .\ 1}{x = a\ .\ .\ .\ .\ .\ x\ x\ x\ x\ x\ x\ .\ .\ .\ .\ .\ b}$$

$$S_2: \quad \frac{y = 0\ .\ .\ .\ .\ .\ 1\ 0\ 1\ 0\ 1\ 1\ .\ .\ .\ .\ .\ 1}{x = a\ .\ .\ .\ .\ .\ x\ x\ x\ x\ x\ x\ .\ .\ .\ .\ .\ b}$$

$$S_3: \quad \frac{y = 0\ .\ .\ .\ .\ .\ 0\ 0\ 1\ 0\ 1\ 0\ .\ .\ .\ .\ .\ 1}{x = a\ .\ .\ .\ .\ .\ x\ x\ x\ x\ x\ x\ .\ .\ .\ .\ .\ b}$$

Figure 6. Primary Region

Once a terminal sequence (S_1, S_2, or S_3) has been obtained, maximum likelihood estimates of the parameters of the stimulus distribution are obtained.

Let n be the number of trials in a terminal sequence. The estimates, $\hat{\mu}$ and $\hat{\sigma}$, are found by maximizing:

$$\prod_{i=1}^{n} \text{Prob}(Y_i = y_i | x_i;\ \hat{\mu},\hat{\sigma}) \qquad \text{Equation (1)}$$

where

$$\text{Prob}(Y_i = y_i | x_i;\ \hat{\mu},\hat{\sigma}) = \begin{cases} N(x_i;\ \hat{\mu},\hat{\sigma}) & \text{if } y_i = 1 \\ 1 - N(x_i;\ \hat{\mu},\hat{\sigma}) & \text{if } y_i = 0 \end{cases}$$

and

$$N(x;\ \hat{\mu},\hat{\sigma}) = \text{Prob}(X \leq x;\ \hat{\mu},\hat{\sigma}) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\hat{\sigma}}\, e^{-\frac{(x-\hat{\mu})^2}{2\hat{\sigma}^2}}\, dx.$$

In other words, given a sequence of responses (y_1, $y_2, \ldots, y_n$) for stimulus levels (x_1, $x_2, \ldots, x_n$), find the maximum likelihood estimates (mle's) of the parameters that describe the normal distribution from which the data was realized. These estimates, $\hat{\mu}$ and $\hat{\sigma}$, can then be used as inputs to the Staircase or Probit techniques. If more trials are desired in order to yield more accurate estimates, then the investigator should follow the extension procedures as described below.

C. EXTENSION OF SEARCH

1. Extension of S_1

For the terminal sequence S_1 the responses are fairly symmetric about $\hat{\mu}$, so that additional trials should be carried out in the primary region. Figure 7 depicts the stimulus levels to test in the event three or five additional trials are desired.

```
 0    0    1    0    1    1
____________________________
        x    x    x

      Three Trials

 0    0    1    0    1    1
____________________________
      x   x   x   x   x

      Five Trials
```

Figure 7. Extension of S_1

2. <u>Extension of S_2 or S_3</u>

It appears that one of the ends of the primary region is fairly well pinned down while the other is not. Therefore additional trials are recommended as shown in Figure 8.

			1	0	1	0	1	1
x	x	x						

S_2 Extension

0	0	1	0	1	0			
						x	x	x

S_3 Extension

Figure 8. Extensions of S_2 and S_3

D. EXAMPLE OF A SEARCH USING MBS

A sample experiment may go as follows. Suppose an investigator is testing for the flashpoint of an explosive. This type of an event certainly qualifies as belonging to the sensitivity class. From past experience he roughly chooses a = 0 $^\circ$C and b = 1000 $^\circ$C. His first trial is conducted at 500 $^\circ$C and he observes that the explosive ignites. The next level (temperature) at which to test the explosive is thus 250 $^\circ$C. Continuing his experiment he arrives at terminal sequence S_1 after eight trials.

Trial	Level ($^\circ$C)	Outcome
1	500	1 (ignition)
2	250	0 (no ignition)
3	750	1 (S*)
4	375	0

Trial	Level (°C)	Outcome
5	625	1 (S*)
6	437.5	0
7	562.5	0 (inversion)
8	687.5	1

An ordered listing of the trial levels and respective outcomes is as follows. The equations for the mle's will appear later.

y =	0	0	0	0	1	0	1	1	1	1
x =	0	250	375	437.5	500	562.5	625	687.5	750	1000 (°C)

The maximum likelihood estimates are: $\hat{\mu} = 531.25\,^{\circ}C$; $\hat{\sigma} = 81.25\,^{\circ}C$.

III. NATURE OF PROBLEM

In order to adequately test the MBS a digital computer was used for both simulation and data processing. It appeared that a standard analytical method (e.g. using derivatives) to locate the maximum of Equation (1) was not feasible. However, a favorable property of Equation (1) is that the function is unimodal over the entire range of $\hat{\mu}$ and $\hat{\sigma}$. Thus, search routines (e.g. Direct, Hookes-Jeeves) can be used to find estimates of the maximum of Equation (1) and the associated $\hat{\mu}$ and $\hat{\sigma}$ as well.

All estimates of threshold parameters calculated in [5] were done by hand, or desk calculator, thus permitting the possibility of significant round off error. The only inputs (response data) to Equation (1) were taken from the primary inversion region, while responses outside this region were ignored. This seemed to have the greatest effect of the sequences S_2 and S_3. Inclusion of all responses was considered reasonable and thus written into the computer program used in this study.

In an earlier study on MBS Hicks [3] claimed that $\hat{\sigma}$ underestimated σ significantly with no predictable bias. The distribution of $\hat{\sigma}$ was therefore investigated in this study for each of the terminal sequences. In addition, the distribution of $\hat{\mu}$ was investigated.

IV. SIMULATION

A. DESCRIPTION

The computer program was written in the Fortran IV programming language and executed on an IBM 360/67 computer. The program was written in two sections, I and II.

Section I conducted a simulated sensitivity experiment leading to a terminal sequence (S_1, S_2, or S_3). A standard normal distribution was used as the source of the sensitivity data. The distribution endpoints (a, b) were varied in order to test MBS over a wide range of starting points. In most cases a~U(-9, -3) and b~U(3, 9). At each stimulus level, x_i, a random number was drawn from the simulated distribution and tested. If the random number was greater than x_i, then $y_i = 0$. The program followed the complete search procedure as illustrated in Figures 2 through 5. A simple flow chart is diagrammed in Figure 9.

Section II took the vectors (x_1, x_2,..., x_n) and (y_1, y_2,..., y_n) from Section I and calculated estimates of μ and σ to within .03 DELX of the values μ^* and σ^* that maximize Equation (1). Initially a "direct" search was employed to locate the mle's for a given sequence. Different guesses of $\hat{\mu}$ and $\hat{\sigma}$ were used as inputs to Equation (1). This method proved to be very inefficient. A Hookes-Jeeves search routine for two variables was tried

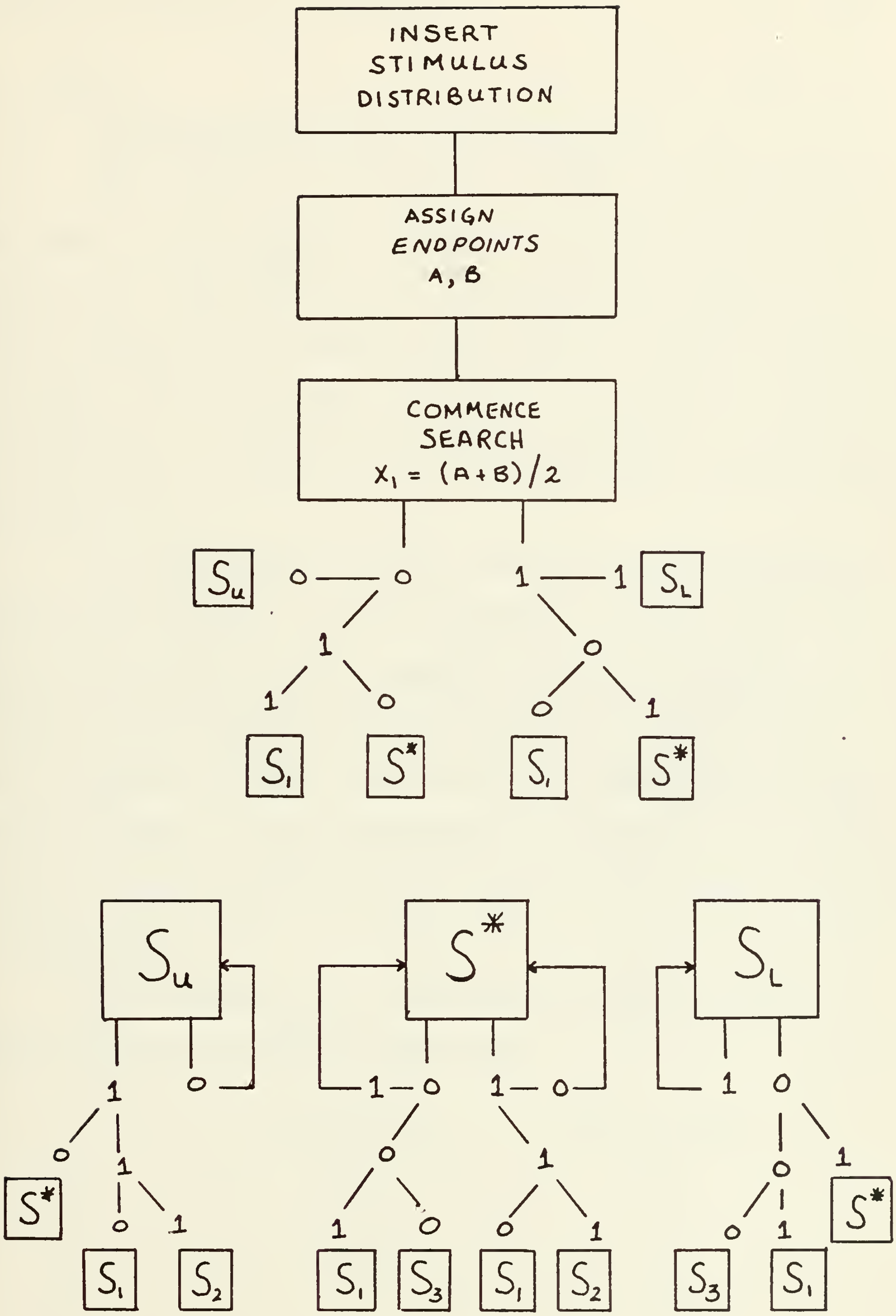

INSERT STIMULUS DISTRIBUTION
ASSIGN ENDPOINTS A, B
COMMENCE SEARCH
$X_1 = (A+B)/2$
S_u
S_L
S_1
S^*
S_2
S_3

FIGURE 9

with much better success (a copy of this routine can be found at the end of the thesis).

B. RESULTS

1. Verification of Previous Estimates

The majority of estimates in [5] were verified by the computer simulations. Table 1 lists the results for comparison. The values of $\hat{\mu}$ are listed in displacement from x_s where x_s is the smallest level at which a positive response occurred. This displacement is given in multiples of DELX (dx). The values of $\hat{\sigma}$ are also given as multiples of DELX (dx).

The estimates, $\hat{\mu}$ and $\hat{\sigma}$, for S_1 are not affected by the total number of levels tested. However, the estimates from an S_2 or S_3 do depend on the total number of trials in the sequence. The values for $\hat{\mu}$ and $\hat{\sigma}$ for S_2 and S_3 listed in Table 1 are valid for sequences of eight or more trials. If an investigator reaches S_2 or S_3 with seven or fewer trials he should extend his search or repeat it altogether.

The frequency of occurrences of S_1, S_2, and S_3 from a sample of 2885 sequences was:

	frequency	percentage
S_1	2226	77.2
S_2	330	11.4
S_3	329	11.4

Sequence	Previous Estimates [5] $\hat{\mu}$	$\hat{\sigma}$	Revised Estimates $\hat{\mu}$	$\hat{\sigma}$
S_1	$x_s + dx/2$	1.3dx	$x_s + .5dx$	1.3dx
S_2	$x_s - dx/4$	6dx	$x_s + 1.2dx$	3dx
S_3	$x_s + 13dx/4$	6dx	$x_s + 1.8dx$	3dx

S_1 extension

(three additional trials)

outcome	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
0 0 0	$x_s + 1.5dx$	1.5dx	$x_s + .9dx$	1.5dx
0 0 1	$x_s + .75dx$	dx	$x_s + .2dx$	dx
0 1 0	$x_s + dx$	2dx	$x_s + .3dx$	1.6dx
0 1 1	$x_s + .25dx$	dx	$x_s - .2dx$	dx
1 0 0	$x_s + dx$	2.75dx	$x_s + .3dx$	2.3dx
1 0 1	x_s	2dx	$x_s - .3dx$	1.6dx
1 1 0	x_s	2.75dx	$x_s - .3dx$	2.3dx
1 1 1	$x_s - dx$	1.5dx	$x_s - .9dx$	1.5dx

$dx \equiv$ DELX

TABLE 1

SIMULATION RESULTS

2. Biased Estimates

Levels tested in the vicinity of the mean of the stimulus distribution offer the most informative data, while those tested within a few standard deviations of the mean aid in pinning down the tails of the distribution. The Probit technique requires the investigation to commence fairly close to the mean. The Probit and Staircase methods use a constant step size between levels and thus perform best when the step size $\approx \sigma$. For instance, if the step size $\gg \sigma$, then an investigator might never test within three or four standard deviations of the mean. If the step size $\ll \sigma$, then one would need many trials to insure that an informative range of levels was tested in the vicinity of the mean. A useful estimate of σ is one that falls within the range of $\sigma/2$ to 2σ.

The values listed in Table 1 are functions of x_s and DELX, or DELX alone. The level x_s (smallest level at which a positive response occurs) is random but tends toward the mean of the distribution. DELX is also random but depends upon the selection of the endpoints, a and b, the threshold variance, and the sequential outcomes $(y_1, y_2, \ldots, y_n)$. Thus DELX can take on values over a wide range implying $\hat{\mu}$ and $\hat{\sigma}$ can as well.

On the basis of substantial sampling, estimates of $\hat{\mu}$ appear to be unbiased for S_1, but seem to have a small predictable bias for S_2 and S_3. The average $\hat{\mu}$ from simulated N(0, 1) sensitivity data was about -0.1 for S_2

and +0.1 for S_3. This would correspond to $\mu - .1\sigma$ and $\mu + .1\sigma$ for N (μ,σ) data.

The estimate $\hat{\sigma}$ is on the other hand not so well behaved. It appears to have unpredictable bias for all sequences (Hicks [3]). Less than half of the estimates of $\hat{\sigma}$ generated from hundreds of simulated trials were within the desired range ($\sigma/2$ to 2σ). The majority were less than $\sigma/2$.

Of primary concern is why the apparent bias? Is Equation (1) in error? Equation (1) does not depend upon the number of trials necessary for an inversion, whereas DELX does. It appears that MBS generates too many trials (i.e. DELX is frequently small in comparison to σ when the first inversion occurs). Hence, the average estimate of $\hat{\sigma}$ is much smaller than the true standard deviation.

Consider an estimate $\hat{\sigma}$ generated from an S_1 sequence using Equation (1). Table 1 lists $\hat{\sigma}$ = 1.3 DELX as the maximum likelihood estimate. An investigator would hope that if the true threshold variance were equal to 1.00, then DELX would be close to .77. If so, then $\hat{\sigma}$ would appear unbiased and qualify as a useful input to follow-up tests. However, the <u>average</u> length of DELX from 1000 S_1 sequences generated from N(0, 1) sensitivity data equalled .49. With an S_2 or S_3 sequence, an investigator would hope that DELX would be close to .33, since $\hat{\sigma}$ = 3 DELX. Again a large sample (675) was investigated with an average DELX of .21. Thus the maximum likelihood principal

(Equation (1)) yielded an average estimate of $\hat{\sigma}$ that was substantially smaller than the true standard deviation. Figures 10 through 12 are histograms of the frequency distributions of DELX for S_1, S_2 and S_3. Table 2 lists the same data.

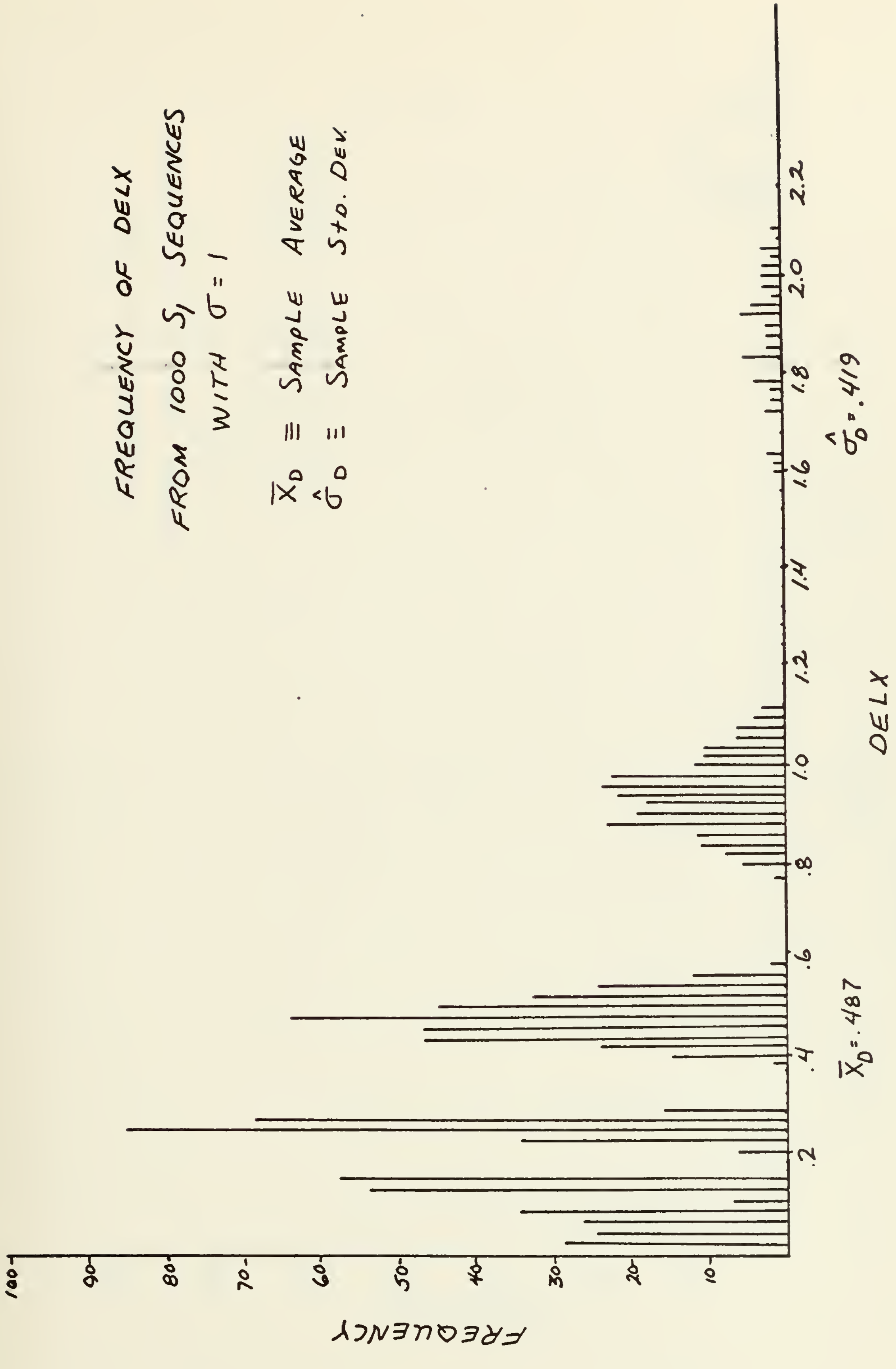
A summary of the associated $\hat{\sigma}$'s from the simulated sequences was as follows:

	$\hat{\sigma} < \sigma/2$	$\sigma/2 < \hat{\sigma} < 2\sigma$	$\hat{\sigma} > 2\sigma$
S_1	45.2%	50.6%	4.2%
S_2	51.4%	47.7%	0.9%
S_3	51.1%	47.4%	1.2%

The estimates of $\hat{\sigma}$ were improved slightly by using an extension of three trials of S_1. The results were as follows.

$\hat{\sigma} < \sigma/2$	$\sigma/2 < \hat{\sigma} < 2\sigma$	$\hat{\sigma} > 2\sigma$
44%	54%	2%

Figure 13 is a histogram of the latter distribution.



FREQUENCY OF DELX
FROM 1000 S_1 SEQUENCES
WITH $\sigma = 1$
$\overline{X}_D \equiv$ SAMPLE AVERAGE
$\hat{\sigma}_D \equiv$ SAMPLE STD. DEV.
$\overline{X}_D = .487$
$\hat{\sigma}_D = .419$
DELX
FREQUENCY
100
90
80
70
60
50
40
30
20
10
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2

FIGURE 10

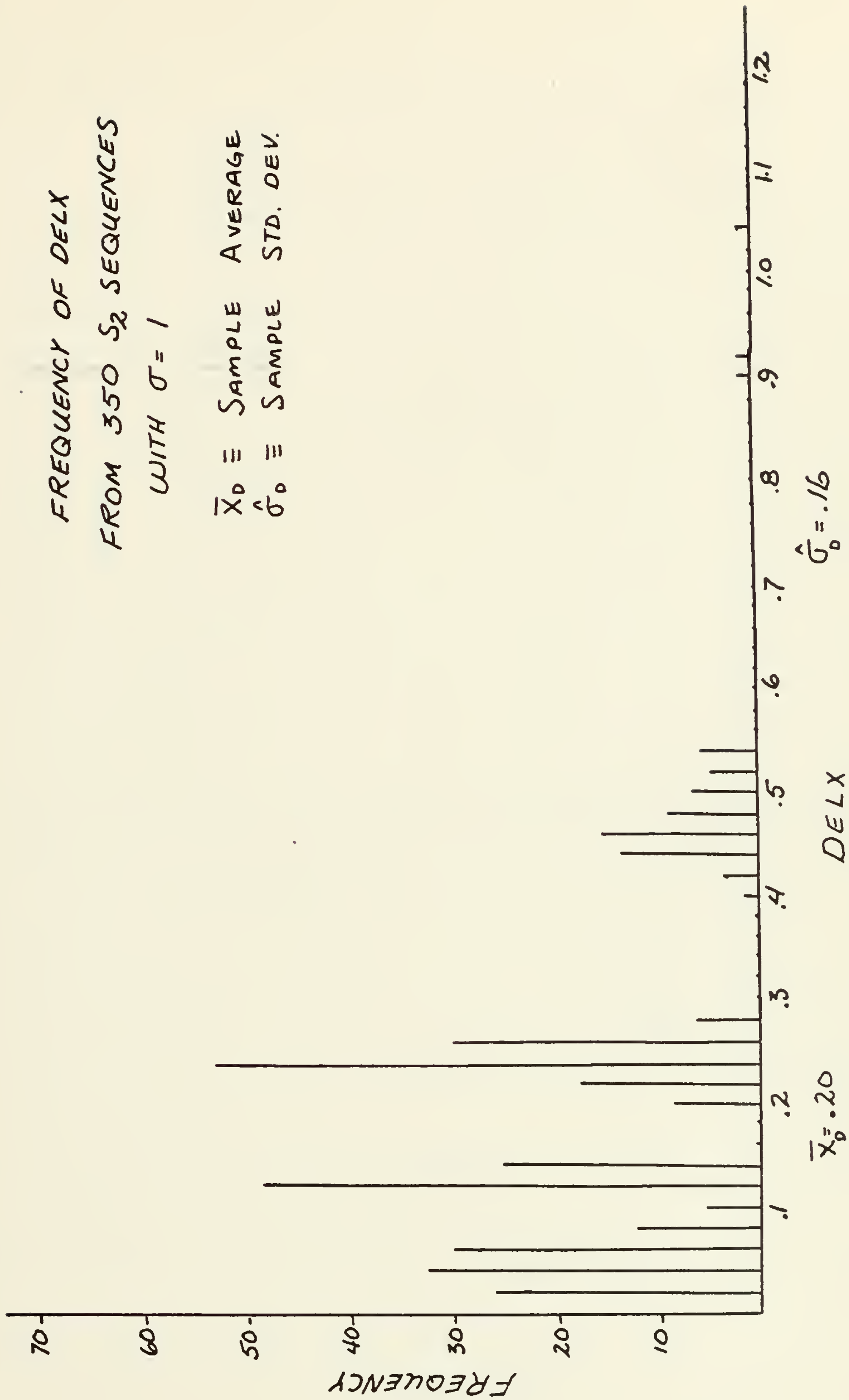
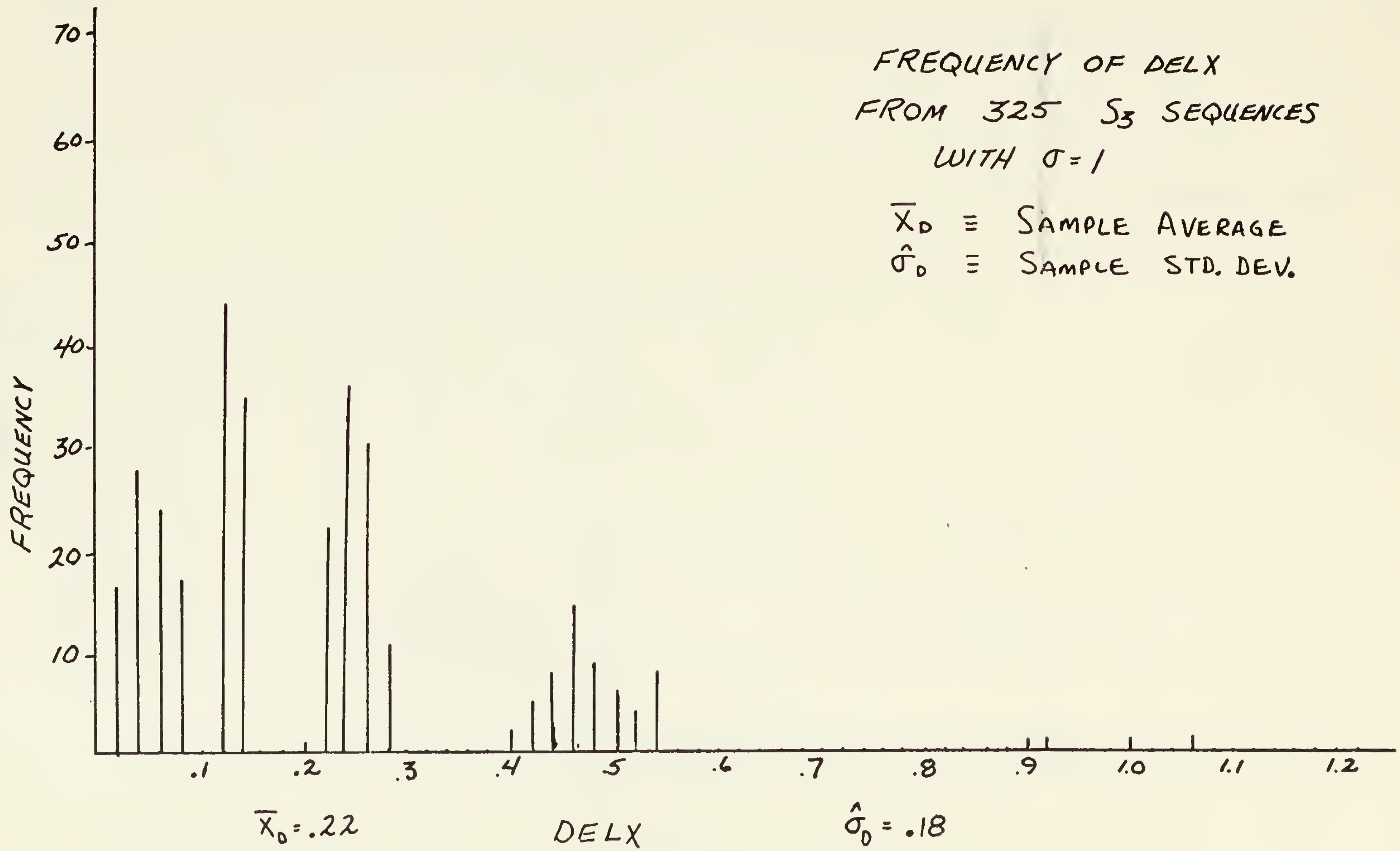

FREQUENCY OF DELX
FROM 350 S_2 SEQUENCES
WITH $\sigma = 1$
$\bar{X}_D \equiv$ SAMPLE AVERAGE
$\hat{\sigma}_D \equiv$ SAMPLE STD. DEV.
FREQUENCY
10
20
30
40
50
60
70
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
$\bar{X}_D = .20$
$\hat{\sigma}_D = .16$
DELX

FIGURE 11



FREQUENCY OF DELX
FROM 325 S_3 SEQUENCES
WITH $\sigma = 1$
$\bar{X}_D$ ≡ SAMPLE AVERAGE
$\hat{\sigma}_D$ ≡ SAMPLE STD. DEV.
FREQUENCY
70
60
50
40
30
20
10
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
$\bar{X}_D = .22$
DELX
$\hat{\sigma}_D = .18$

FIGURE 12

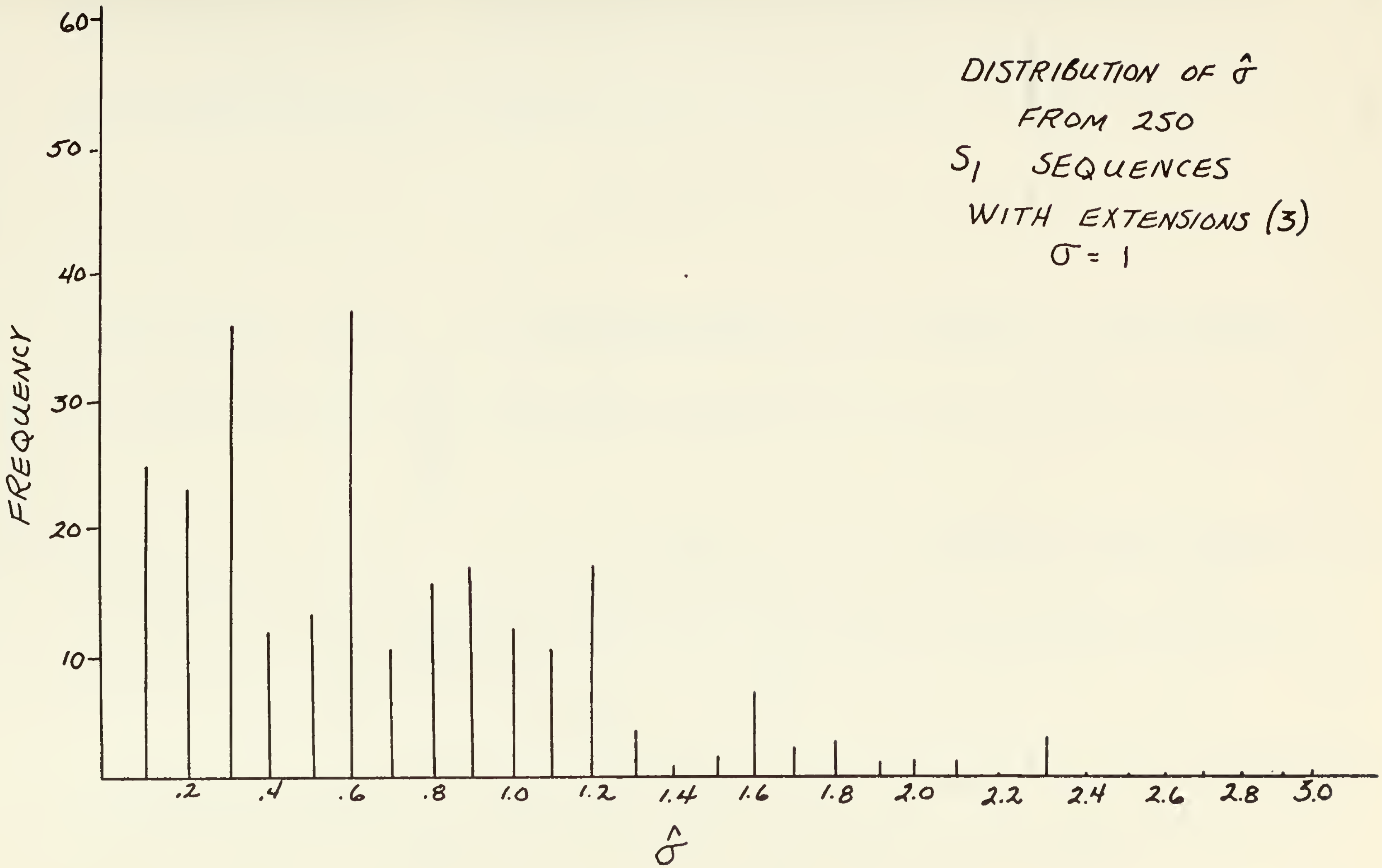

DISTRIBUTION OF $\hat{\sigma}$
FROM 250
S_1 SEQUENCES
WITH EXTENSIONS (3)
$\sigma = 1$
FREQUENCY
60
50
40
30
20
10
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
$\hat{\sigma}$

FIGURE 13

RANGE OF DELX	S_1		S_2		S_3	
.00 to .02	29		26		17	
.02 to .04	24		32		28	
.04 to .06	26		30		24	
.06 to .08	34		14	$\hat{\sigma}<\sigma/2$	18	$\hat{\sigma}<\sigma/2$
.08 to .10	7		5		0	
.10 to .12	53		48		43	
.12 to .14	57		25		35	
.14 to .16	0		0		0	
.16 to .18	0	$\hat{\sigma}<\sigma/2$	0		0	
.18 to .20	6		8		1	
.20 to .22	35		17		22	
.22 to .24	84		52		36	
.24 to .26	68		29		30	
.26 to .28	17		6		11	
.28 to .30	0		0		0	
.30 to .32	0		0		0	
.32 to .34	0		0		0	
.34 to .36	0		0		0	
.36 to .38	1		0		0	
.38 to .40	13		1	$\sigma/2<\hat{\sigma}<2\sigma$	2	$\sigma/2<\hat{\sigma}<2\sigma$
.40 to .42	24		3		5	
.42 to .44	46		13		8	
.44 to .46	46		15		14	
.46 to .48	63		8		9	
.48 to .50	44		6		6	
.50 to .52	32		4		4	
.52 to .54	23		5		8	
.54 to .56	12		0		0	
.56 to .58	1		0		0	
.58 to .60	0		0		0	
. . .	.		.		.	
. . .	.	$\sigma/2<\hat{\sigma}<2\sigma$	.		.	
. . .	.		.		.	
.72 to .74	0		0		0	
.74 to .76	1		0		0	
.76 to .78	0		0		0	
.78 to .80	6		0		0	
.80 to .82	8		0		0	
.82 to .84	11		0		0	
.84 to .86	13		0		0	
.86 to .88	23		0		0	
.88 to .90	19		1		1	
.90 to .92	18		1		1	
.92 to .94	21		0		0	
.94 to .96	23		0		0	

TABLE 2

FREQUENCY DISTRIBUTION OF DELX

STIMULUS ~ N(0,1)

RANGE OF DELX	S_1		S_2	S_3
.96 to .98	20		0	0
.98 to 1.00	11		0	0
1.00 to 1.02	10		0	0
1.02 to 1.04	10		.	.
1.04 to 1.06	6		.	.
1.06 to 1.08	6		.	.
1.08 to 1.10	4			
1.10 to 1.12	3			
1.12 to 1.14	0			
1.14 to 1.16	0			
. . .	.			
. . .	.			
. . .	.			
1.56 to 1.58	0			
1.58 to 1.60	1			
1.60 to 1.62	1		$\hat{\sigma}>2\sigma$	$\hat{\sigma}>2\sigma$
1.62 to 1.64	2			
1.64 to 1.66	0			
1.66 to 1.68	0			
1.68 to 1.70	2			
1.70 to 1.72	1			
1.72 to 1.74	1			
1.74 to 1.76	1			
1.76 to 1.78	3			
1.78 to 1.80	0			
1.80 to 1.82	0			
1.82 to 1.84	5	$\hat{\sigma}>2\sigma$		
1.84 to 1.86	2			
1.86 to 1.88	2			
1.88 to 1.90	2			
1.90 to 1.92	5			
1.92 to 1.94	3			
1.94 to 1.96	1			
1.96 to 1.98	2			
1.98 to 2.00	2			
2.00 to 2.02	2			
2.02 to 2.04	1			
2.04 to 2.06	2		.	.
2.06 to 2.08	0		.	.
2.08 to 2.10	1		.	.
2.10 to 2.12	0		0	0
2.12 --	0		0	0
TOTAL	1000		350	325

TABLE 2 (Continued)

V. CONCLUSIONS AND RECOMMENDATIONS

Estimates of μ using MBS are suitable for follow-up tests while those of σ are not. Additional trials will be necessary to refine the estimate of σ.

Termination of MBS with a small number of trials cannot be guaranteed. It is not possible for an investigator to determine beforehand the number of trials necessary to complete a terminal situation (S_1, S_2, or S_3). The average number generated in this investigation was eleven with extreme values of five and twenty-eight. It appears that this number is a function of the location of endpoints, the variance of the threshold distribution, and the sequential outcomes of the experiment.

The extension procedure in MBS should be investigated so that levels yielding the most useful data can be tested. For example, the middle level tested in an S_1 extension doesn't offer data to confirm or alter previous estimates of μ and σ whereas other levels may do so significantly.

As an alternative approach to obtain practical estimates of σ, a Monte Carlo method is suggested in place of the maximum likelihood principle. Note that sample values of DELX are "bunched" together in distinct intervals separated by intervals in which no value of DELX occurs (see Table 2). If an investigator could determine the interval to which the DELX from his experiment belongs, then he should be able to estimate σ better than before.

COMPUTER SEARCH FOR MLES

THE FOLLOWING PROGRAM FINDS THE MAXIMUM LIKELIHOOD ESTIMATES OF THE MEAN AND STANDARD DEVIATION OF ANY SEQUENCE OR EXTENSION.

PSI(1) IS THE INITIAL ESTIMATE OF THE MEAN.

PSI(2) IS THE INITIAL ESTIMATE OF THE STD. DEVIATION.

DELLC IS THE DESIRED ACCURACY OF THE MLES.

MAXEV IS THE MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR ANY SEARCH.

DELCAP IS THE INITIAL STEP SIZE.

RHO IS THE REDUCTION FACTOR OF THE STEP SIZE ONCE A LOCAL MAXIMUM HAS BEEN FOUND.

```
883 PSI(1)=T(NL)+DELX
885 PSI(2)=2.5*DELX
    DELCAP=DELX
    DELLC=.05*DELX
    RHO=.125
    MAXEV=75
    SLC(1)=DELCAP
    SLC(2)=DELCAP
    SPSI=S(PSI(1),PSI(2))
    IVAL=1
801 SS=SPSI
    PHI(1)=PSI(1)
    PHI(2)=PSI(2)
    IBK=-1
    GO TO 840
813 IF(SS.LT.SPSI) GO TO 803
802 IF(IVAL.GE.MAXEV) GO TO 852
    DO 820 K=1,2
    IF(SLC(K)) 821,852,822
821 IF(PHI(K).LE.PSI(K)) SLC(K)=-SLC(K)
    GO TO 823
822 IF(PHI(K).GE.PSI(K)) SLC(K)=-SLC(K)
823 THET = PSI(K)
    PSI(K) = PHI(K)
    PHI(K) = 2.*PHI(K) - THET
820 CONTINUE
    IF(PHI(2)) 815,815,816
815 PHI(2)=PSI(2)/2.
816 SPSI = SS
    SPHI=S(PHI(1),PHI(2))
    SS=SPHI
    IVAL=IVAL + 1
    IBK=1
840 DO 841 K=1,2
    THET = PHI(K)
    SLCI = SLC(K)
    PHI(K) = THET + SLCI
    IF(K-2) 824,825,825
825 IF(PHI(2)) 829,829,824
829 PHI(2)=THET/2.
824 SPHI=S(PHI(1),PHI(2))
    IVAL = IVAL + 1
    IF(SPHI.GE.SS) GO TO 842
    PHI(K) = THET - SLCI
    IF(K-2) 818,819,819
819 IF(PHI(2)) 817,817,818
```

```
817 PHI(2) = THET/2.
818 SPHI=S(PHI(1),PHI(2))
    IVAL = IVAL + 1
    IF( SPHI.LT.SS) GO TO 844
    SLC(K)=-SLCI
842 SS=SPHI
    GO TO 841
844 PHI(K) = THET
841 CONTINUE
    IF(IBK) 813,813,828
828 IF(SS.LT.SPSI) GO TO 801
    DO 826 K=1,2
    IF(ABS(PHI(K)-PSI(K)).GT.0.5*ABS(SLC(K))) GO TO 802
826 CONTINUE
    DELF=DELCAP
803 IF(DELCAP.LT.DELLC) GO TO 852
    DELCAP=RHO*DELCAP
    SLC(1)=RHO*SLC(1)
    SLC(2)=RHO*SLC(2)
    GO TO 801
```

STATEMENT 852 IS THE EXIT FROM THE ROUTINE.

PHI(1) IS THE FINAL ESTIMATE OF THE MEAN.

PHI(2) IS THE FINAL ESTIMATE OF THE STD. DEVIATION.

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13. ABSTRACT

A modified binary search routine was developed by Tysver to provide inputs (estimates of parameters of threshold stimulus distributions) to the Probit and Staircase techniques for sensitivity testing. A high speed digital computer was used to provide simulated sensitivity data in order to test the modified search (more specifically, study the parameter estimates generated by the search).

The results show that the modified search does not give practical input information to the techniques mentioned above. However, further refinements of extensions to the basic search patterns should yield better estimates of the distribution parameters.

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